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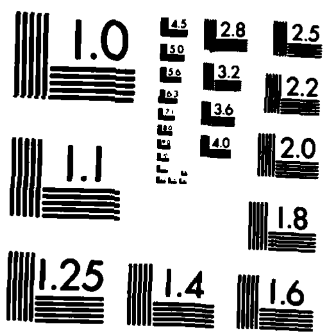

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GEOPOTENTIAL HARMONICS OF ORDER 15 AND 30, FROM ANALYSIS OF THE ORBIT OF SATELLITE 1971-10B

by

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UNLIMITED

UDC 521.3 : 629.19.077.3 : 521.6 : 517.564.4

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SUMMARY

The satellite 1971-10B passed through exact 15th-order resonance on 1981 March 30, and orbital parameters have been determined at 52 epochs from some 3500 observations using the RAE orbit refinement program, PROP, between 1980 September and 1981 October.

The variations in inclination and eccentricity during this time have been analysed, and six lumped 15th-order harmonic coefficients and two 30th-order coefficients have been evaluated. The 15th-order coefficients are the best yet obtained for an orbital inclination near 65°; and previously there were no 30th-order coefficients available at this inclination.

The lumped coefficients have been used to test the Goddard Earth Model GEM 10B: there is good agreement for seven of the eight coefficients.

Departmental Reference: Space 633

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LIST OF CONTENTS

1 INTRODUCTION

Cosmos 394 rocket, designated 1971-10B, entered orbit on 1971 February 9 and initially had the following orbital elements[1]: inclination 65.84°; nodal period 96.43 min; apogee and perigee heights, 612 km and 564 km respectively; and eccentricity 0.003. The satellite will decay in 1984.

By 1981 March the orbital period of the satellite had decreased to 95 min and the orbit was passing through the condition of 15th-order resonance. This occurs when the track over the Earth repeats after 15 revolutions, *ie* when the satellite makes 15 revolutions while the Earth spins once relative to the orbital plane. For this Report the orbit of the satellite has been determined from radar and visual observations between 1980 September and 1981 October, using the RAE orbit refinement program PROP in the PROP 6 version[2]. Then the inclination and eccentricity were analysed over this time to evaluate lumped harmonic coefficients of order 15 and 30 in the geopotential.

The analysis of this orbit at 15th-order resonance is of particular interest because of its inclination, 65.8°. In previous evaluations of the individual 15th-order harmonic coefficients in the geopotential[3] from the lumped harmonic coefficients derived from the analysis of satellite orbits at various inclinations, there have been no accurate lumped coefficients available for an inclination near 65°.

2 THE OBSERVATIONS, ORBITS AND OBSERVATIONAL ACCURACY

2.1 Observations

The orbit of 1971-10B has been determined from some 3500 observations, between 1980 September 3 and 1981 October 28, at 52 epochs. The observations used fell into three groups and a breakdown of the number and type used on each orbit is given in Table 1.

The first group consists of visual observations made by volunteer observers who report to the University of Aston in Birmingham and these observations usually have accuracies between 1 and 4 minutes of arc. The second group is made up of Navspasur observations kindly supplied by the US Naval Research Laboratory, with accuracies of about 2 minutes of arc. The third group comprises kinetheodolite observations made at the South African Astronomical Observatory, accurate to about 1 minute of arc. Observations from the kinetheodolite were only available on orbits 1-18, as the instrument closed down at the end of January 1981.

2.2 The orbits

The orbits were determined at approximately 8-day intervals from 1980 September to 1981 October with the aid of the RAE orbit refinement program in the PROP 6 version, and the orbital elements at each of the 52 epochs are listed in Table 2, with the standard deviations below each value. The epoch for each orbit is at 00 hours on the day indicated, and the PROP program fits the mean anomaly M by a polynomial of the form

$$M = M_0 + M_1 t + M_2 t^2 + M_3 t^3 + M_4 t^4 + M_5 t^5 \quad , \qquad (1)$$

where t is the time measured from epoch, and the number of M coefficients used depends

Table 1

Sources of observations used in each orbit

Orbit No.	Source of observation				Orbit No.	Source of observation		
	Visual	US Navy	Cape kine	Total		Visual	US Navy	Total
1		51		51	27	17	76	93
2	8	78		86	28	1	75	76
3	7	61		68	29	4	59	63
4	9	74		83	30		55	55
5	8	59		67	31	1	46	47
6	7	61		68	32	4	41	45
7	6	73		79	33	7	71	78
8		45	3	48	34	4	88	92
9	8	66		74	35		65	65
10	1	42	1	44	36	5	59	64
11	3	69	2	75	37	6	50	56
12	4	84		88	38		69	69
13		49		49	39		56	56
14	16	54		70	40	4	62	66
15	8	55		63	41		62	62
16	18	80		98	42		54	54
17	4	64		68	43	11	53	64
18	1	55	9	65	44	18	67	85
19	3	47		50	45	57	35	92
20	6	48		54	46	4	53	57
21		48		48	47	4	50	54
22		86		86	48	15	56	71
23	2	113		115	49	7	45	52
24	6	68		74	50	4	63	67
25	3	75		78	51		78	78
26	10	80		90	52	3	44	47

on the drag. For 1971-10B, which was in a nearly circular orbit at about 500 km height, $M_0 - M_3$ were the coefficients required on 30 orbits; but for the remaining 22 orbits only the coefficients $M_0 - M_2$ were needed.

The value of ε, the parameter indicating the measure of fit of the observations to the orbit, varied between 0.33 and 1.16, and had an average value of 0.62.

For all 52 orbits the standard deviations in inclination were between 0.0007° and 0.0024°, the average being 0.0013° which is equivalent to about 150 m in distance. The values of eccentricity had an average standard deviation of 0.000013 over the 52 orbits, equivalent to 90 m in distance. The individual standard deviations in e varied between 0.000006 and 0.000027. The right ascension of the node had nearly the same accuracy as the inclination.

2.3 The accuracy of the observations

The residuals of the observations have been obtained using the ORES computer program[4] and sent to the observers. Table 3 lists the accuracies of the observing stations with five or more observations accepted in the orbit determinations. The Table shows the number of observations used from a particular station, divided into those

Table 2

Values of the orbital parameters at 52 epochs with standard deviations

MJD	Date	a	e	i	Ω	ω	ω + M_0	M_1	M_2	M_3	ε	N	D	a(1 − e)
1 44485	3 Sep 1980	6911.5617	0.002650	65.8353	314.750	63.8	48.589	5440.2205	0.0244	0.00098	0.33	42	7.4	6893.25
		5	6	8	1	2		6	1	5				
2 44493	11 Sep 1980	6911.1563	0.002677	65.8341	290.114	58.5	7.325	5440.6993	0.0316	0.00046	0.58	67	7.4	6892.66
		6	10	12	1	3		7	1	8				
3 44501	19 Sep 1980	6910.7448	0.002632	65.8316	265.469	54.4	330.038	5441.1852	0.0283	-	0.47	57	7.9	6892.56
		3	8	9	1	2		3	1					
4 44510	28 Sep 1980	6910.2390	0.002582	65.8308	237.738	50.9	337.706	5441.7826	0.0413	0.00108	0.74	76	7.9	6892.40
		6	13	13	1	3		8	2	8				
5 44519	7 Oct 1980	6909.4839	0.002547	65.8332	209.999	44.9	352.011	5442.6752	0.0574	-	0.66	60	7.6	6891.89
		2	14	13	1	3		3	2					
6 44527	15 Oct 1980	6908.6158	0.002502	65.8352	185.336	40.5	332.481	5443.7014	0.0635	-0.00065	0.67	57	7.6	6891.33
		6	15	9	1	3		7	2	7				
7 44535	23 Oct 1980	6907.8094	0.002529	65.8327	160.657	34.9	321.104	5444.6546	0.0542	-	0.71	60	7.7	6890.34
		4	16	16	1	3		5	2					
8 44549	6 Nov 1980	6906.5908	0.002623	65.8307	117.450	23.7	227.286	5446.0959	0.0602	-	0.60	38	7.9	6888.47
		3	9	12	1	3		4	2					
9 44557	14 Nov 1980	6905.7622	0.002715	65.8338	92.749	18.8	235.067	5447.0766	0.0579	-	0.70	66	7.4	6887.01
		3	16	15	1	2		4	2					
10 44565	22 Nov 1980	6905.0594	0.002730	65.8358	68.033	13.6	250.253	5447.9085	0.0451	-0.00049	0.46	38	7.3	6886.21
		8	14	12	1	2		9	2	10				
11 44575	2 Dec 1980	6904.1937	0.002665	65.8370	37.135	8.5	8.049	5448.9336	0.0556	-	0.55	53	7.4	6885.79
		2	14	11	1	2		3	1					
12 44583	10 Dec 1980	6903.5135	0.002686	65.8322	12.401	4.6	37.916	5449.7388	0.0521	0.00096	0.51	69	7.4	6884.97
		5	13	9	1	1		6	1	7				
13 44591	18 Dec 1980	6902.6971	0.002605	65.8324	347.659	359.2	74.589	5450.7058	0.0717	0.00118	0.68	46	8.9	6884.72
		8	20	14	1	2		10	2	7				
14 44600	27 Dec 1980	6901.6784	0.002538	65.8333	319.807	353.8	171.175	5451.9130	0.0596	-	0.47	55	8.2	6884.16
		3	14	9	1	2		3	1					
15 44609	5 Jan 1981	6900.7976	0.002494	65.8337	291.948	350.0	277.651	5452.9571	0.0477	-0.00182	0.88	56	7.6	6883.59
		10	27	19	2	3		12	2	12				
16 44617	13 Jan 1981	6900.2888	0.002543	65.8330	267.174	345.2	338.878	5453.5603	0.0355	-	0.64	90	7.7	6882.74
		2	14	9	1	2		2	1					
17 44625	21 Jan 1981	6899.8693	0.002601	65.8311	242.393	340.1	44.537	5454.0577	0.0279	-	0.44	56	7.9	6881.92
		2	12	11	1	1		3	1					
18 44633	29 Jan 1981	6899.4373	0.002634	65.8292	217.607	333.9	113.951	5454.5699	0.0410	0.00060	0.53	54	7.9	6881.26
		6	7	9	1	2		7	1	7				
19 44641	6 Feb 1981	6898.8083	0.002644	65.8301	192.814	329.9	188.440	5455.3161	0.0538	0.00078	0.59	43	7.4	6880.57
		9	16	14	1	2		11	2	11				

Table 2 (continued)

MJD	Date	a	e	i	Ω	ω	ω + M_0	M_1	M_2	M_3	ε	N	D	a(1 − e)
20 44649	14 Feb 1981	6898.1561	0.002623	65.8332	168.012	324.7	269.343	5456.0902	0.0425	–	0.49	48	7.4	6880.06
		3	13	11	1	2		3	2					
21 44657	22 Feb 1981	6897.5682	0.002687	65.8347	143.206	321.6	355.865	5456.7880	0.0465	–	0.58	37	7.4	6879.03
		4	17	16	1	3		5	2					
22 44665	2 Mar 1981	6896.8209	0.002661	65.8343	118.390	318.6	88.605	5457.6752	0.0622	0.00085	0.52	61	7.4	6878.47
		6	13	10	1	2		7	2	8				
23 44673	10 Mar 1981	6895.8646	0.002605	65.8299	93.561	314.8	189.590	5458.8105	0.0669	–	0.54	82	7.9	6877.90
		2	10	10	1	2		2	1					
24 44681	18 Mar 1981	6894.8663	0.002587	65.8287	68.717	310.5	299.745	5459.9962	0.0750	–	0.73	69	6.4	6877.03
		4	13	14	1	3		5	2					
25 44689	26 Mar 1981	6893.8650	0.002512	65.8329	43.863	303.2	59.416	5461.1864	0.0858	0.00111	0.77	69	7.5	6876.55
		9	14	19	2	4		10	2	11				
26 44697	3 Apr 1981	6892.5087	0.002513	65.8341	18.995	298.9	190.103	5462.7989	0.1058	−0.00084	0.72	78	7.6	6875.19
		8	12	15	1	4		10	2	9				
27 44705	11 Apr 1981	6891.1131	0.002526	65.8358	354.114	293.5	334.056	5464.4590	0.1174	0.00178	1.16	76	7.6	6873.71
		14	16	16	2	6		17	2	15				
28 44713	19 Apr 1981	6889.7276	0.002554	65.8330	329.209	292.3	131.952	5466.1076	0.0870	–	0.69	73	7.6	6872.13
		3	9	13	1	4		4	2					
29 44721	27 Apr 1981	6888.6940	0.002526	65.8314	304.289	288.9	301.050	5467.3380	0.0698	–	0.62	53	8.0	6871.29
		3	10	14	1	4		4	2					
30 44729	5 May 1981	6887.7904	0.002529	65.8301	279.356	286.3	119.073	5468.4141	0.0707	0.00053	0.48	46	7.4	6870.37
		8	9	11	1	3		9	1	8				
31 44737	13 May 1981	6886.7326	0.002512	65.8326	254.412	282.0	306.362	5469.6745	0.0725	−0.00129	0.57	41	7.5	6869.43
		9	11	16	1	4		11	2	11				
32 44745	21 May 1981	6885.8966	0.002524	65.8335	229.457	276.9	143.038	5470.6710	0.0461	−0.00160	0.51	42	7.7	6868.52
		7	9	11	1	4		9	1	8				
33 44753	29 May 1981	6885.3574	0.002559	65.8318	204.496	270.4	346.116	5471.3136	0.0339	–	0.51	62	7.4	6867.74
		3	7	12	1	3		3	2					
34 44761	6 Jun 1981	6884.9476	0.002569	65.8299	179.525	266.8	193.765	5471.8021	0.0236	–	0.60	82	7.9	6867.26
		2	6	10	1	3		3	1					
35 44769	14 Jun 1981	6884.6344	0.002505	65.8293	154.549	261.4	44.733	5472.1755	0.0239	0.00055	0.44	55	8.9	6867.25
		4	6	8	1	3		5	1	4				
36 44778	23 Jun 1981	6884.1913	0.002542	65.8321	126.446	256.1	331.006	5472.7042	0.0294	−0.00038	0.52	46	7.6	6866.69
		8	10	12	1	3		9	1	8				
37 44786	1 Jul 1981	6883.8049	0.002580	65.8332	101.460	250.5	189.598	5473.1652	0.0278	–	1.00	45	7.6	6866.04
		6	19	23	2	7		7	3					
38 44794	9 Jul 1981	6883.4934	0.002561	65.8340	76.473	246.1	51.525	5473.5369	0.0210	–	0.55	67	7.6	6865.86
		2	9	12	1	3		3	1					
39 44803	18 Jul 1981	6883.1108	0.002516	65.8324	48.353	241.9	349.644	5473.9932	0.0318	0.00042	0.48	48	9.9	6865.79
		5	9	11	1	3		6	1	4				

80078

Table 2 (concluded)

MJD	Date	a	e	i	Ω	ω	ω + M_0	M_1	M_2	M_3	ε	N	D	a(1 - e)
40 44813	28 Jul 1981	6882.4452 7	0.002517 13	65.8303 14	17.100 1	235.8 3	7.011	5474.7874 8	0.0393 2	-0.00143 6	0.71	52	8.9	6865.12
41 44821	5 Aug 1981	6882.0095 3	0.002541 11	65.8307 15	352.093 1	231.1 3	242.463	5475.3074 4	0.0302 2	-	0.58	53	7.5	6864.52
42 44829	13 Aug 1981	6881.5748 6	0.002475 11	65.8337 10	327.081 1	226.6 3	121.892	5475.8266 7	0.0415 1	0.00093 7	0.49	48	7.5	6864.54
43 44837	21 Aug 1981	6880.8621 7	0.002418 12	65.8339 9	302.061 1	222.8 2	6.671	5476.6776 8	0.0666 1	0.00144 7	0.48	53	7.6	6864.22
44 44845	29 Aug 1981	6879.8279 8	0.002442 14	65.8302 12	277.030 1	218.2 3	259.919	5477.9125 9	0.0847 2	0.00041 9	0.67	68	7.8	6863.03
45 44851	4 Sep 1981	6879.0386 7	0.002442 26	65.8294 16	258.245 2	214.1 5	6.604	5478.8555 8	0.0734 7	-	0.99	80	3.9	6862.24
46 44857	10 Sep 1981	6878.2539 5	0.002417 11	65.8285 7	239.457 1	209.0 2	118.761	5479.7933 5	0.0875 1	0.00190 5	0.40	45	7.8	6861.63
47 44865	18 Sep 1981	6876.9882 9	0.002527 18	65.8288 15	214.384 1	203.5 3	38.179	5481.3066 11	0.0958 2	0.00043 10	0.60	46	7.6	6859.61
48 44873	26 Sep 1981	6875.7338 10	0.002513 21	65.8329 17	189.301 2	198.0 3	329.735	5482.8072 12	0.0952 2	0.00079 11	0.77	59	7.6	6858.46
49 44881	4 Oct 1981	6874.4072 8	0.002546 23	65.8304 18	164.204 1	192.5 3	273.487	5484.3946 10	0.0979 2	-0.00142 10	0.73	43	7.6	6856.90
50 44889	12 Oct 1981	6873.0931 4	0.002438 19	65.8301 17	139.088 1	189.2 2	229.785	5485.9679 5	0.1013 2	-	0.66	53	7.0	6856.34
51 44897	20 Oct 1981	6871.6064 9	0.002298 22	65.8274 17	113.952 1	185.6 2	199.336	5487.7486 11	0.1334 1	0.00258 8	0.78	57	7.9	6855.82
52 44905	28 Oct 1981	6869.8566 9	0.002177 18	65.8264 14	88.792 1	180.4 2	185.067	5489.8457 11	0.1174 2	-0.00054 12	0.54	41	6.8	6854.90

KEY

MJD	modified Julian day
a	semi major axis (km)
e	eccentricity
i	inclination (deg)
Ω	right ascension of ascending node (deg)
ω	argument of perigee (deg)
M_0	mean anomaly at epoch (deg)
M_1	mean motion n (deg/day)
M_2,M_3	*later coefficients in the polynomial for M*
ε	measure of fit
N	number of observations used
D	time covered by the observations (days)

accepted and rejected in the orbit determinations. The US Navy observations from station 29 are geocentric, and if they were given in the same form as the topocentric observations, their angular rms residuals would increase by a factor of about five.

The residuals for station 29 were similar to their usual standard. Those for stations 1–6 were slightly larger, but this is because low-elevation observations (<20°) were retained with their assumed error doubled. The kinetheodolite observations had an average accuracy of 1.8 minutes of arc and the visual observations averaged about 5 minutes of arc. These accuracies are slightly worse than usual, for no obvious reason.

Table 3

Residuals for observing stations with five or more observations accepted in the final orbit determination

Station	Number of observations		Rms residuals			
	Accepted	Rejected	Range km	Minutes of arc		
				RA	Dec	Total
1 US Navy	315	47		2.6	2.9	3.9
2 US Navy	216	32		3.3	3.0	4.5
3 US Navy	166	23		4.3	3.7	5.7
4 US Navy	186	33		4.0	3.2	5.1
5 US Navy	228	21		2.6	2.8	3.3
6 US Navy	266	19		2.7	2.1	3.4
29 US Navy	1314	35	0.6	0.3	0.4	
2183 Pengwern	6	1		12.7	9.2	15.7
2265 Farnham	14	3		4.5	6.3	7.7
2392 Cowbeech	5	0		1.4	2.2	2.6
2414 Bournemouth	56	0		3.2	3.4	4.7
2418 Sunningdale	23	1		4.1	4.1	5.8
2420 Willowbrae	85	29		3.5	2.6	4.4
2577 Cape kinetheodolite	12	3		1.3	1.2	1.8
4156 Apeldoorn	9	1		2.9	2.8	4.1

3 THE EQUATIONS AT 15TH-ORDER RESONANCE

The theory for general $\beta:\alpha$ resonance has been given before, in Ref 3, where all parameters used in the following equations are defined.

The theoretical equations at 15th-order resonance for the variation of inclination and eccentricity are as follows:

$$\frac{di}{dt} = \frac{n}{\sin i}\left(\frac{R}{a}\right)^{15}\Bigg[(15 - \cos i)\bar{F}_{15,15,7}\left\{\bar{C}_{15}^{0,1}\sin\Phi - \bar{S}_{15}^{0,1}\cos\Phi\right\}$$

$$+ \frac{17e}{2}(15)\left(\frac{R}{a}\right)\bar{F}_{16,15,8}\left\{\bar{S}_{15}^{1,0}\sin(\Phi - \omega) + \bar{C}_{15}^{1,0}\cos(\Phi - \omega)\right\}$$

$$+ \frac{13e}{2}(15 - 2\cos i)\left(\frac{R}{a}\right)\bar{F}_{16,15,7}\left\{\bar{S}_{15}^{-1,2}\sin(\Phi + \omega) + \bar{C}_{15}^{-1,2}\cos(\Phi + \omega)\right\}$$

$$+ \text{terms in}\left\{\frac{(\frac{1}{2}\ell e)^{|q|}}{(|q|)!}\,\begin{matrix}\cos\\ \sin\end{matrix}(\gamma\Phi - q\omega)\right\}\Bigg] \quad , \qquad (2)$$

and

$$\frac{de}{dt} = \frac{n}{2}\left(\frac{R}{a}\right)^{15}\Bigg[\!\!\Bigg[e\bar{F}_{15,15,7}\left(\bar{C}_{15}^{0,1}\sin\Phi - \bar{S}_{15}^{0,1}\cos\Phi\right)$$

$$- 17\left(\frac{R}{a}\right)\bar{F}_{16,15,8}\left\{\bar{S}_{15}^{1,0}\sin(\Phi - \omega) + \bar{C}_{15}^{1,0}\cos(\Phi - \omega)\right\}$$

$$+ 13\left(\frac{R}{a}\right)\bar{F}_{16,15,7}\left\{\bar{S}_{15}^{-1,2}\sin(\Phi + \omega) + \bar{C}_{15}^{-1,2}\cos(\Phi + \omega)\right\}$$

$$+ \text{terms in}\left[\frac{(\frac{1}{2}\ell)^{|q|}e^{|q|-1}}{(|q|)!}\left\{q - \tfrac{1}{2}(k + q)e^2\right\}\begin{matrix}\cos\\ \sin\end{matrix}(\gamma\Phi - q\omega)\right]\!\!\Bigg]\!\!\Bigg] \quad . \qquad (3)$$

The resonance angle Φ is given by

$$\Phi = \omega + M + 15(\Omega - \nu) \quad , \qquad (4)$$

where ν is the sidereal angle, and at exact resonance $\dot{\Phi} = 0$. In equations (2) and (3) only the three terms with $(\gamma,q) = (1,0)(1,1)$ and $(1,-1)$ are given explicitly.

The three pairs of lumped coefficients in equations (2) and (3) can be expressed as linear sums of individual 15th-order coefficients

$$\bar{C}_m^{q,k} = \sum Q_\ell^{q,k}\bar{C}_{\ell m} \quad \text{and} \quad \bar{S}_m^{q,k} = \sum Q_\ell^{q,k}\bar{S}_{\ell m} \quad , \qquad (5)$$

and these are given explicitly, with the $Q_\ell^{q,k}$ expressed in terms of the $\bar{F}$ inclination functions, in Ref 3.

The orders of magnitude of the (γ,q) terms in the equation for di/dt can be estimated, as the $\bar{C}_{\ell m}$ (or $\bar{S}_{\ell m}$) are expected to be of order $10^{-5}/\ell^2$, so the value of $\bar{C}_m^{q,k}$ (or $\bar{S}_m^{q,k}$) can be taken to be of order $\left\{\sum(Q_\ell \times 10^{-5}/\ell^2)^2\right\}^{\frac{1}{2}}$. The F and Q coefficients can be calculated for specified values of (γ,q) using the computer program

PROF. The expected orders of magnitude are given in Table 4 without the factor

$$\frac{n}{\sin i}\left(\frac{R}{a}\right)^{15}(15 - \cos i) \times 10^{-9} .$$

Table 4

Estimated orders of magnitude of terms in equation for di/dt

q \ γ	1	2	3
0	19	2.4	0.4
+1	0.4	0.1	-
-1	0.3	0.1	-

The expected orders of magnitude for the (γ,q) terms in the equation for de/dt can be estimated in a similar manner to di/dt and they are given in Table 5 without the factor $n\left(\frac{R}{a}\right)^{15} \times 10^{-9}$.

Table 5

Estimated orders of magnitude of terms in equation for de/dt

q \ γ	1	2	3
0	0.02	-	-
+1	157	21	4
-1	136	18	3
+2	3	-	-
-2	2	-	-

The orders of magnitude of the (γ,q) terms in Tables 4 and 5 suggest that taking the $(\gamma,q) = (1,0)(2,0)$ terms for inclination will only introduce an error of 2% or less, and taking the $(\gamma,q) = (1,\pm1)(2,\pm1)$ terms for eccentricity only 3% or less.

4 THE ANALYSIS OF INCLINATION AT 15TH-ORDER RESONANCE

The satellite 1971-10B passed through exact 15th-order resonance on 1981 March 30. The variation of the resonance angle Φ , given by equation (4), and the rate of change of the resonance angle, $\dot{\Phi}$, which increased from -18 to +18 deg/day between 1980 September and 1981 October, are shown in Fig 1.

Before the changes in inclination due to resonance can be analysed, all other known perturbations must be removed. The 52 values of inclination available in Table 2 were cleared of lunisolar and zonal harmonic perturbations, by using the computer program PROD[5] with 1-day integration steps, and the perturbations due to the $J_{2,2}$ tesseral

84008

harmonic, which is recorded on each PROP run, were also removed. Perturbations due to earth and ocean tides[6] should not exceed 50 m and need not be considered here as the values of inclination have average standard deviations of 150 m.

The values of inclination, cleared of the above perturbations, and with the standard deviations quoted in Table 2, were fitted with equation (2) in integrated form using the THROE[7] computer program which removes perturbations due to atmospheric rotation and lunisolar precession of the Earth's axis. The density scale height H was taken as 73 km, appropriate to a height of 512 km, 0.2H above perigee[8], and the atmospheric rotation rate, Λ, was taken[9] as 0.8 rev/day. The values of M_2 were altered to mean values, $\bar{M}_2$, by the technique described in Ref 10.

The first THROE fitting of i, with the $(\gamma,q) = (1,0)$ terms only, gave a value of ε, the parameter indicating the measure of fit, of 0.726 and the values of the lumped 15th-order harmonics were:

$$10^9\bar{C}_{15}^{0,1} = 2.6 \pm 3.9 \qquad 10^9\bar{S}_{15}^{0,1} = 3.1 \pm 0.9 \quad . \tag{6}$$

The values of these coefficients were surprisingly small: the $10^{-5}/\ell^2$ rule indicated that they should be of order 81×10^{-9}. This means that the $(\gamma,q) = (1,0)$ terms in Table 4 are only about 1/30 of the expected value: it was, therefore, necessary to include the (2,0) terms in the fitting. The $(\bar{C},\bar{S})_{30}^{0,2}$ coefficients in the (2,0) terms should be of order 52×10^{-9} according to the $10^{-5}/\ell^2$ rule and as the $(\bar{C},\bar{S})_{15}^{0,1}$ terms are so small, the (2,0) terms may be dominant.

The second fitting, with $(\gamma,q) = (1,0)(2,0)$ terms included, gave:

$$\left.\begin{aligned} 10^9\bar{C}_{15}^{0,1} &= -0.7 \pm 4.1 & \qquad 10^9\bar{S}_{15}^{0,1} &= 2.4 \pm 3.9 \\ 10^9\bar{C}_{30}^{0,2} &= -54 \pm 27 & \qquad 10^9\bar{S}_{30}^{0,2} &= 59 \pm 40 \end{aligned}\right\} \tag{7}$$

with $\varepsilon = 0.705$. Since the values of $\bar{C}_{30}^{0,2}$, $\bar{S}_{30}^{0,2}$ are slightly greater than the $10^{-5}/\ell^2$ rule and the $\bar{C}_{15}^{0,1}$, $\bar{S}_{15}^{0,1}$ are only 0.03 times the expected value, the relative magnitudes of the (1,0) and (2,0) terms in equation (7) are not 19 and 2.4 respectively, as in Table 4, but 0.6 and 2.7. Thus the (2,0) term is the more important and the solution for (1,0) alone, equation (6), cannot be accepted.

The values of inclination were next fitted with $(\gamma,q) = (1,0)(2,0)$ and (3,0). This fitting was not so good: the (3,0) terms were undetermined and the standard deviations on the other four coefficients were increased. Omitting the small (1,0) terms did not improve the (3,0) terms, as they were still undetermined. A fitting with just $(\gamma,q) = (2,0)$, whilst giving slightly lower standard deviations, was not considered appropriate as the $\bar{C}_{15}^{0,1}$, $\bar{S}_{15}^{0,1}$ coefficients, although small, have standard deviations of the same magnitude as 15th-order coefficients at other inclinations[3] and therefore provide useful information on 15th-order harmonics. So the values in equation (7) were preferred.

The effects of changing the values of H and Λ were investigated. The alteration of H had no effect on the value of ε or the lumped coefficients. Changing the value of Λ to 0.7 or 0.9 increased the value of ε, thus confirming the choice of the value 0.8. The effect of shortening the run was also investigated, by dropping the first five and the last five values of inclination. This resulted in a value of ε only slightly lower, and the values of the harmonic coefficients were not significantly altered. As the change in the inclination at 15th-order resonance is very small, it seemed preferable to keep as many points as possible on either side of exact resonance to firmly establish the values of inclination before and after resonance. So the coefficients obtained using all 52 values of inclination were preferred.

The values of inclination, cleared of all known perturbations except those due to resonance, are plotted in Fig 2, with the theoretical curve derived from the THROE fitting with the $(\gamma,q) = (1,0)(2,0)$ terms, which gave the coefficients in equation (7).

Although the change in inclination as a result of passing through resonance is very small, 0.0010°, Fig 2 shows that the fitting of the values is completely satisfactory. The values of the 15th-order lumped coefficients in equation (7) may look inaccurate, but their standard deviations correspond to errors in geoid height of only about 2 cm.

5 THE ANALYSIS OF ECCENTRICITY AT 15TH-ORDER RESONANCE

The perturbations in eccentricity due to zonal harmonics and atmospheric drag, and the (much smaller) lunisolar perturbations, all have to be removed before the changes due to resonance can be analysed.

The removal of the zonal harmonic and lunisolar perturbations was performed by using the PROD computer program. The variation of e with time on a drag-free orbit was calculated by numerical integration at 1-day intervals with the aid of this program, and this drag-free orbit gives the changes in e due to zonal harmonic and lunisolar perturbations. In Fig 3a the values of eccentricity from Table 2 are plotted as circles and the triangles show the values of e after removal of zonal harmonic and lunisolar perturbations, $e - \Delta e_{ZHL}$. The remaining variation, indicated by the triangles, is due to air drag and resonance.

The air drag model within THROE takes no account of the day-to-night variation in density, which is important for 1971-10B. So the correction due to drag in an atmosphere with day-to-night variation was calculated using the theory given in the Appendix of Ref 11, with scale height H taken as 74 km, appropriate to a mean height of 521 km, which is 0.2H above perigee[12]. This theory assumes that the atmospheric density depends on the geocentric angular distance from the point of maximum density, taken to be on the equator at either 14 h local time (the time indicated by[13] *CIRA 1972*) or at 16 h local time (the time indicated in Jacchia's 1977 model[14]). The correction Δe_D for drag with day-to-night variation is shown in Fig 3b for local times of 14 h and 16 h.

The 52 values of eccentricity from Table 2 could then be cleared of perturbations due to zonal harmonics, lunisolar forces and atmospheric drag, and the resulting values of e should show a variation due to resonance alone. These values of e were then

84008

fitted with equation (3) in integrated form, using THROE. The program was run with no adjustment being made to e to compensate for zonal harmonic and atmospheric perturbations as the values of e had already been cleared of these perturbations.

The first THROE run was with the values of e changed on the assumption that the point of maximum density was at 14 h local time and the second run at 16 h local time. The first run gave $\varepsilon = 3.61$ and the second $\varepsilon = 2.98$; so the values of e adjusted for 16 h local time were adopted. The last six values of e were ill-fitting, however, and when they were omitted, a further fit with THROE gave a value of ε reduced by 7% to 2.76. These fittings were with $(\gamma, q) = (1,1)(1,-1)$.

Previously[15], when fitting values of eccentricity with THROE at inclinations near the critical inclination of 63.4° it was found that some adjustment of the zonal harmonics was needed. This arose because, when the set of odd zonal harmonics used in THROE were evaluated[16], there were only a few satellites available for analysis with inclinations in this critical region. Therefore the values of the odd zonal harmonics used in THROE may not be adequately modelling the variation in eccentricity for 1971-10B, which is at an inclination of 65.83°. The alterations were made by adding an increment ΔJ_3 to J_3, which has a value of -2.531×10^{-6} in the THROE model. Fig 4 shows the variation of ε with ΔJ_3. The lowest value of ε, 2.751, was attained with $\Delta J_3 \simeq -0.013$, and this was accepted as the best fitting. This implies that the lumped harmonic Y defined in Ref 17 is in error by 0.013: since the errors in the values of Y implicit in THROE[16] at nearby inclinations are all at least 0.03, this refinement is well within the limits of error of the model.

This final fitting of the values of eccentricity with THROE (with $\Delta J_3 = -0.013$) yielded values of the lumped harmonics as follows:

$$\left.\begin{array}{llll} 10^9\bar{C}_{15}^{1,0} = & 35.1 \pm 11.7 & 10^9\bar{S}_{15}^{1,0} = & -13.8 \pm 11.0 \\ 10^9\bar{C}_{15}^{-1,2} = & -20.0 \pm 10.7 & 10^9\bar{S}_{15}^{-1,2} = & -18.9 \pm 10.3 \end{array}\right\} . \qquad (8)$$

The values of eccentricity, cleared of all known perturbations except those due to resonance, are plotted as circles in Fig 5 with the standard deviations from Table 2. The theoretical curve is that of the final THROE run, which gave the coefficients in equation (8). A fitting of e by THROE with $(\gamma, q) = (1,\pm 1)$ and $(2,\pm 1)$ terms was also tried, but this resulted in a larger value of ε and undetermined coefficients.

The fitting of eccentricity in Fig 5 is far from satisfactory, as the high value of ε (2.75) indicates. Between the initial point at MJD 44485 and MJD 44625 there appear to be 1½ cycles of a sinusoidal oscillation in the observational values, with period about 85 days and amplitude 0.00008 (*ie* about 500 m). Fig 3b shows that the day-to-night variation in drag has a similar periodicity, but its effects are too small to cancel completely the large oscillation visible in the observational values, which has an amplitude of about 0.00012 (*ie* 800 m). The explanation of this anomaly remains uncertain, but it seems likely to be attributable to inadequate modelling of the day-to-night variation in air density, perhaps due to asymmetry about the equator at times near the solstices, or perhaps resulting from a real atmospheric anomaly between 1980 September and 1981 January.

Whatever the cause of the anomaly in e, it is fortunate that the fitted curve cuts through the mean of the oscillation, and should, therefore, still provide reliable values of the coefficients, even if the standard deviations are disappointingly large.

6 INCLINATION AND ECCENTRICITY FITTED SIMULTANEOUSLY

The values of inclination and eccentricity fitted separately by THROE can be fitted simultaneously using the computer program SIMRES[18]. This program combines the results from a number of THROE runs (each with the same set of (γ,q) terms), and produces a single set of coefficients to fit the data. For this SIMRES fitting the results of THROE runs with $(\gamma,q) = (1,0)(1,1)(1,-1)$ and $(2,0)$ were used. The SIMRES program allows a choice of weighting, so that the contributing THROE runs can be given more or less weight according to their accuracy of fit, which is indicated by the value of ε.

The THROE fitting of inclination with $(\gamma,q) = (1,0)(1,1)(1,-1)$ and $(2,0)$ gave $\varepsilon = 0.602$, and for eccentricity the value of ε was 2.843, when fitted with the same terms. For the SIMRES fitting, therefore, the weighting of e was down-graded by a factor equal to the ratio of the values of ε on the THROE fittings, namely 4.723 $(= 2.843/0.602)$. The values of the lumped coefficients given by this SIMRES fitting are:

$$\left.\begin{array}{llll}
10^9\bar{C}_{15}^{0,1} = 0.9 \pm 3.8 & & 10^9\bar{S}_{15}^{0,1} = 2.7 \pm 3.6 \\
10^9\bar{C}_{15}^{1,0} = 36 \pm 13 & & 10^9\bar{S}_{15}^{1,0} = -14 \pm 12 \\
10^9\bar{C}_{15}^{-1,2} = -18 \pm 12 & & 10^9\bar{S}_{15}^{-1,2} = -18 \pm 12 \\
10^9\bar{C}_{30}^{0,2} = -56 \pm 25 & & 10^9\bar{C}_{30}^{0,2} = 60 \pm 37
\end{array}\right\} . \qquad (9)$$

The fittings, if shown pictorially in Figs 2 and 5, would be indistinguishable from the curves given by the THROE fittings.

The lumped coefficients obtained from the fitting of i and e separately, given in equations (7) and (8) are very similar to those obtained from the simultaneous fitting given in equation (9). So either set of values could be used in future determinations of the individual 15th-order harmonic coefficients in the geopotential.

7 THE EQUATIONS FOR THE INDIVIDUAL COEFFICIENTS

The lumped coefficients can be expressed as linear sums of individual 15th-order coefficients, see equation (5), and may be written

$$\bar{C}_{15}^{0,1} = \bar{C}_{15,15} + Q_{17}^{0,1}\bar{C}_{17,15} + Q_{19}^{0,1}\bar{C}_{19,15} + \ldots \qquad (10)$$

$$\bar{C}_{15}^{1,0} = \bar{C}_{16,15} + Q_{18}^{1,0}\bar{C}_{18,15} + Q_{20}^{1,0}\bar{C}_{20,15} + \ldots \qquad (11)$$

$$\bar{C}_{15}^{-1,2} = \bar{C}_{16,15} + Q_{18}^{-1,2}\bar{C}_{18,15} + Q_{18}^{-1,2}\bar{C}_{20,15} + \ldots \tag{12}$$

$$\bar{C}_{30}^{0,2} = \bar{C}_{30,30} + Q_{32}^{0,2}\bar{C}_{32,30} + Q_{34}^{0,2}\bar{C}_{34,30} + \ldots \tag{13}$$

and similarly for S, on replacing C by S throughout.

The *Q coefficients* in equations (10) to (13) can be evaluated using the computer program PROF, and numerical versions of equations (10) to (13) are given below. In equations (10) *to* (12) the Q terms included go as far as $Q_{39}^{0,1}$, $Q_{38}^{1,0}$ and $Q_{38}^{-1,2}$ respectively, to correspond with Ref 3, and in equation (13) as far as $Q_{42}^{0,2}$, to correspond with Ref 19.

$$\begin{aligned}\bar{C}_{15}^{0,1} = \bar{C}_{15,15} &- 1.830\bar{C}_{17,15} - 0.350\bar{C}_{19,15} + 0.741\bar{C}_{21,15} + 0.701\bar{C}_{23,15} + 0.107\bar{C}_{25,15} \\ &- 0.348\bar{C}_{27,15} - 0.387\bar{C}_{29,15} - 0.140\bar{C}_{31,15} + 0.121\bar{C}_{33,15} + 0.215\bar{C}_{35,15} \\ &+ 0.136\bar{C}_{37,15} - 0.007\bar{C}_{39,15}\end{aligned} \tag{14}$$

$$\begin{aligned}\bar{C}_{15}^{1,0} = \bar{C}_{16,15} &- 1.138\bar{C}_{18,15} - 0.551\bar{C}_{20,15} + 0.478\bar{C}_{22,15} + 0.698\bar{C}_{24,15} + 0.206\bar{C}_{26,15} \\ &- 0.333\bar{C}_{28,15} - 0.461\bar{C}_{30,15} - 0.196\bar{C}_{32,15} + 0.152\bar{C}_{34,15} + 0.303\bar{C}_{36,15} \\ &+ 0.199\bar{C}_{38,15}\end{aligned} \tag{15}$$

$$\begin{aligned}\bar{C}_{15}^{-1,2} = \bar{C}_{16,15} &- 0.085\bar{C}_{18,15} - 0.635\bar{C}_{20,15} - 0.479\bar{C}_{22,15} + 0.009\bar{C}_{24,15} + 0.372\bar{C}_{26,15} \\ &+ 0.385\bar{C}_{28,15} + 0.129\bar{C}_{30,15} - 0.156\bar{C}_{32,15} - 0.272\bar{C}_{34,15} - 0.183\bar{C}_{36,15} \\ &+ 0.005\bar{C}_{38,15}\end{aligned} \tag{16}$$

$$\begin{aligned}\bar{C}_{30}^{0,2} = \bar{C}_{30,30} &- 3.971\bar{C}_{32,30} + 3.166\bar{C}_{34,30} + 1.636\bar{C}_{36,30} - 1.323\bar{C}_{38,30} - 1.607\bar{C}_{40,30} \\ &- 0.149\bar{C}_{42,30} ,\end{aligned} \tag{17}$$

and similarly for S, on replacing C by S throughout.

8 LUMPED COEFFICIENTS FROM 1971-10B COMPARED WITH THOSE OBTAINED FROM GEM 10B

The values of individual coefficients from the comprehensive geopotential model, Goddard Earth Model GEM 10B [20], which extends to degree and order 36, may be substituted into equations (14)-(17) and the corresponding S equations to give lumped coefficients.

Table 6

Values of lumped coefficients

	$10^9\bar{C}^{0,1}_{15}$	$10^9\bar{S}^{0,1}_{15}$	$10^9\bar{C}^{1,0}_{15}$	$10^9\bar{S}^{1,0}_{15}$	$10^9\bar{C}^{-1,2}_{15}$	$10^9\bar{S}^{-1,2}_{15}$	$10^9\bar{C}^{0,2}_{30}$	$10^9\bar{S}^{0,2}_{30}$
1971-10B	-1 ± 4	2 ± 4	35 ± 12	-14 ± 11	-20 ± 11	-19 ± 10	-54 ± 27	59 ± 40
GEM 10B	5 ± 7	3 ± 7	71 ± 6	-12 ± 6	-8 ± 4	-29 ± 4	-47 ± 16	14 ± 16

The 15th- and 30th-order lumped coefficients thus obtained from GEM 10B are given in Table 6, together with those from the THROE fittings of inclination and eccentricity for 1971-10B. In Table 6 the accuracy of the GEM 10B individual coefficients is assumed[3] to be 3×10^{-9}, and the accuracies for the lumped coefficients have been calculated on this assumption.

GEM 10B emerges well from this test of its accuracy by a completely independent method. The two sets of values agree well: seven of the eight coefficients differ by less than the sum of their standard deviations; for the eighth coefficient ($\bar{C}^{1,0}_{15}$) the values differ by twice the sum of their standard deviations.

It is not fair to compare the values obtained from 1971-10B with those from other comprehensive models, such as GRIM 3[21] or the 1981 model by R.H. Rapp[22] at Ohio State University, because both these models incorporated 15th-order terms from Ref 3 in their solutions. The values of lumped coefficients derived from satellites at inclinations near 65° which were used in Ref 3 were known to be of poor accuracy, so any comparison with the two models mentioned would only show where the solution of Ref 3 was in error. A more fruitful procedure is to use these values from 1971-10B for revising Ref 3; and this has now been done[23].

9 CONCLUSIONS

The orbit of 1971-10B has been determined at 52 epochs, from some 3500 observations, during the time when the orbit was affected by 15th-order resonance. The average accuracy of inclination and eccentricity on the 52 orbits was equivalent to 150 m and 90 m in distance respectively.

The variations in inclination and eccentricity have been analysed: six 15th-order and two 30th-order lumped harmonic coefficients have been evaluated. The six 15th-order lumped coefficients have already been used in new determinations of the individual 15th-order harmonic coefficients in the geopotential from lumped harmonic coefficients derived from the analysis of satellite orbits at various inclinations[23]. Previously there were no accurate lumped coefficients available at an inclination near 65° and the addition of the results from 1971-10B has greatly improved the solutions.

Though the values obtained here for 1971-10B are better than any previous results for inclination near 65°, it is hoped to determine still better values, especially for even-degree coefficients, from the satellite 1965-14A which has recently been passing through 15th-order resonance at a slower rate.

The two 30th-order lumped coefficients should be very valuable in a determination of the individual 30th-order coefficients, because previously there were no results available near 65° inclination. This expectation has already been confirmed[23].

The eight lumped coefficients obtained from 1971-10B have been used to test GEM 10B: good agreement was obtained.

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84008

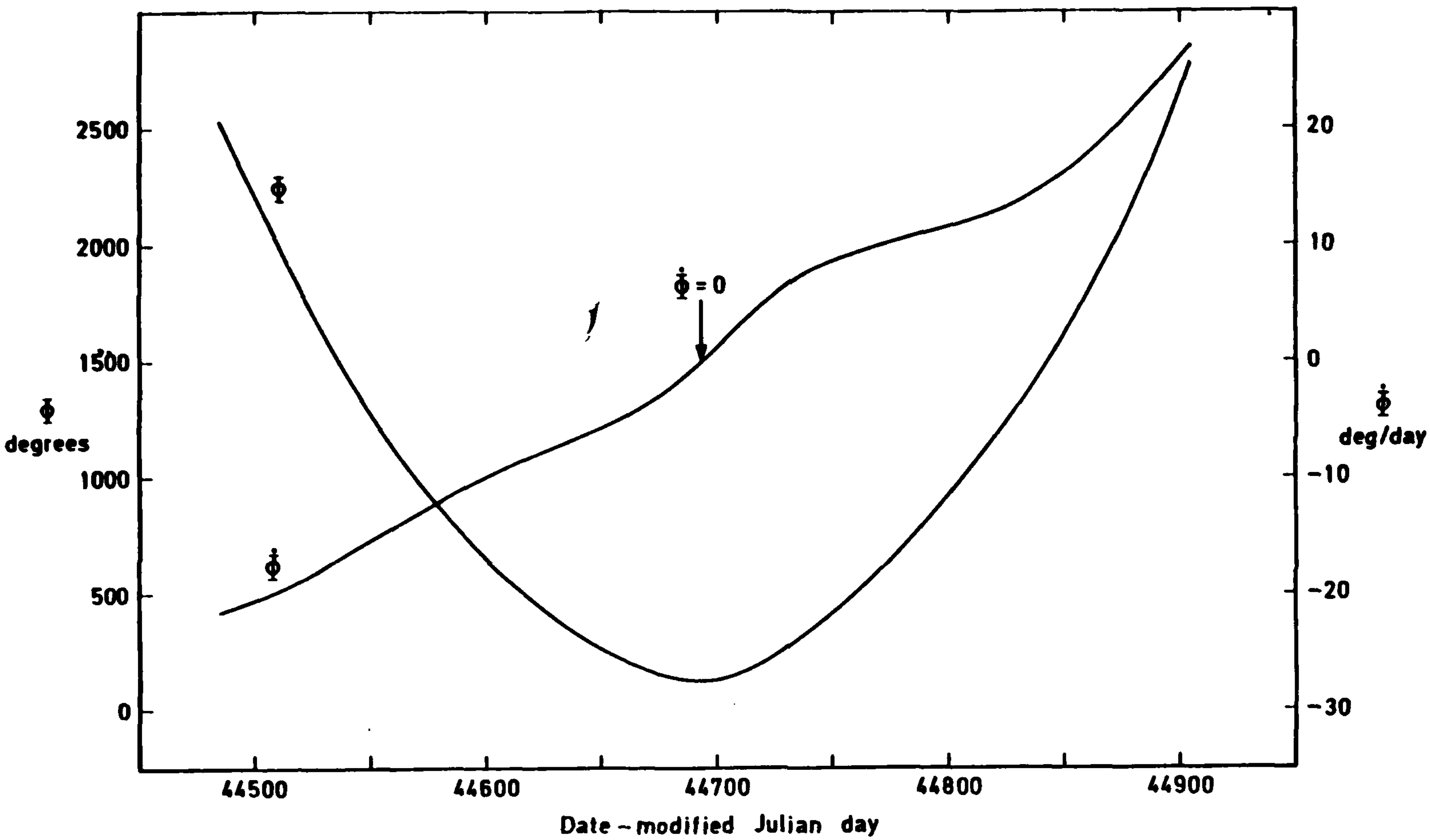

Fig 1 Variation of Φ and $\dot{\Phi}$ near 15th-order resonance

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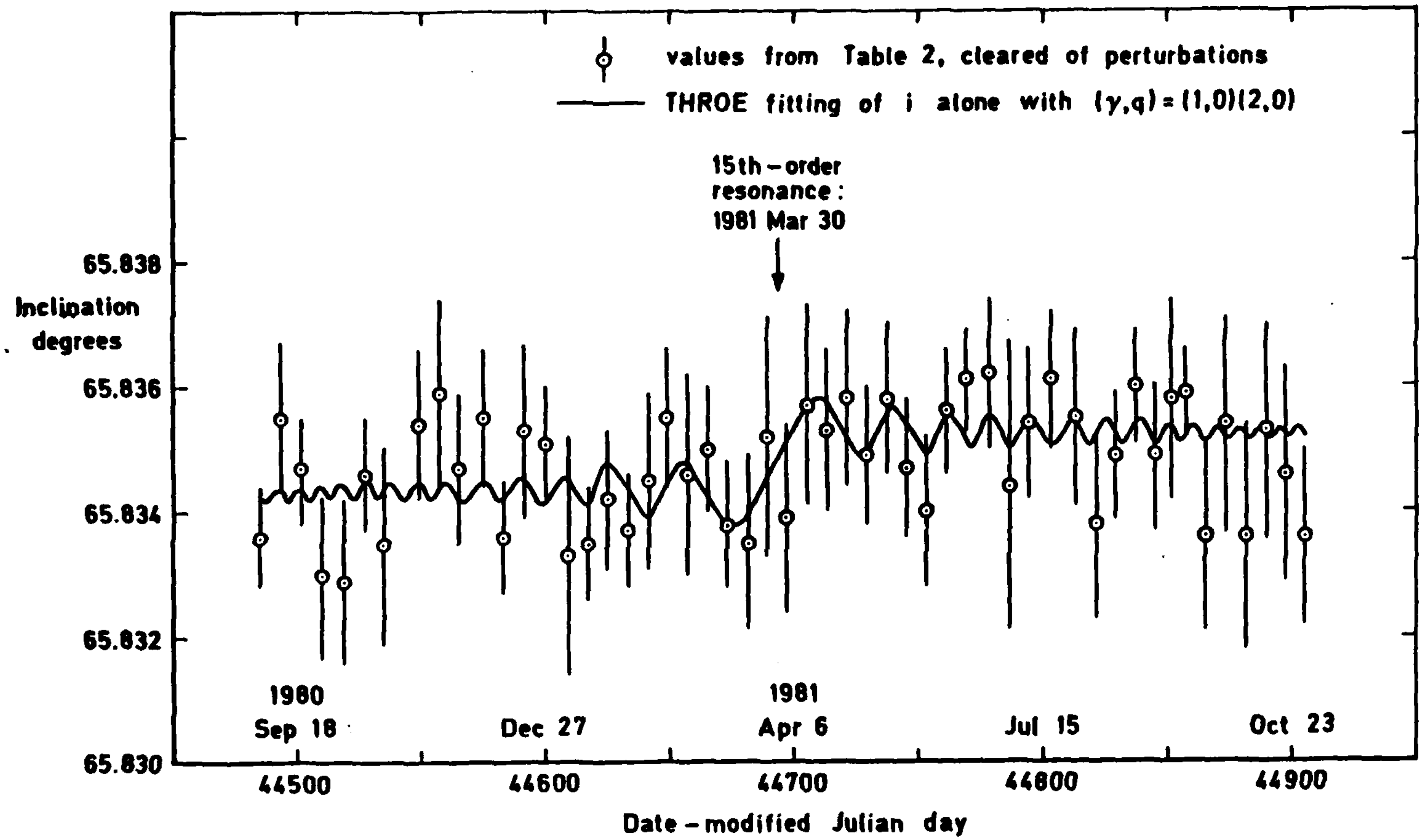

Fig 2 Values of inclination near 15th-order resonance, with fitted theoretical curve

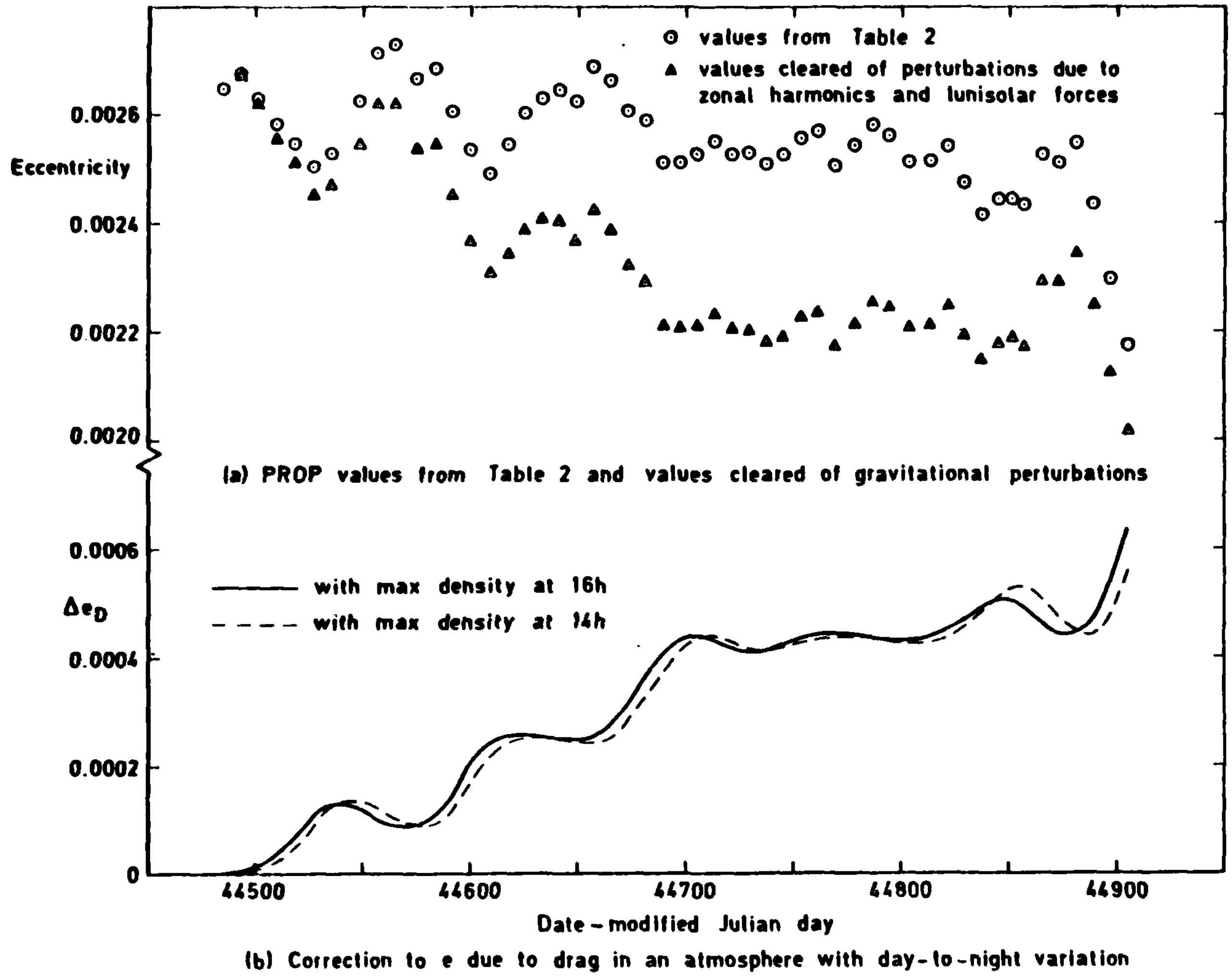

Fig 3a&b Values of eccentricity and perturbations

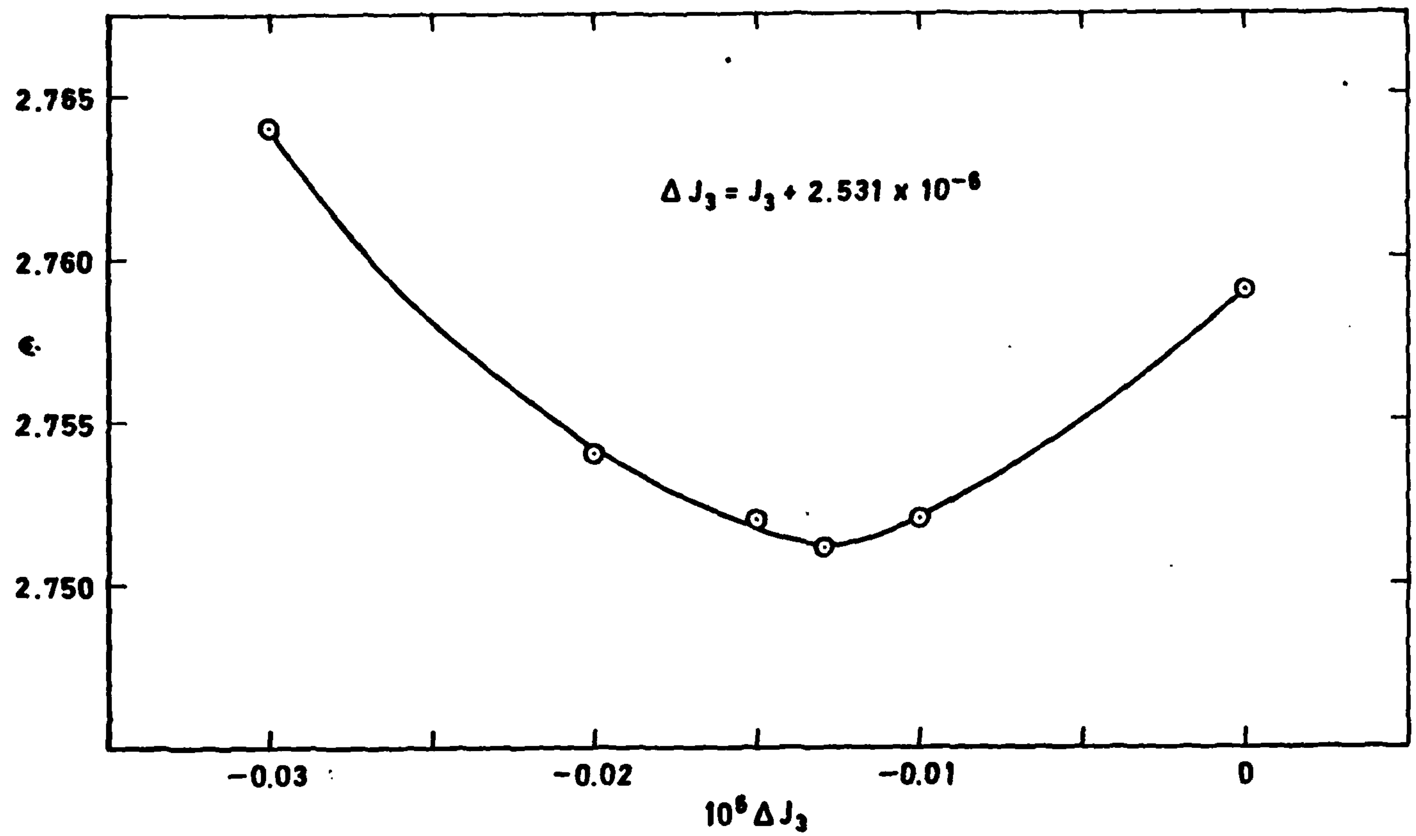

Fig 4 Variation of ϵ with ΔJ_3

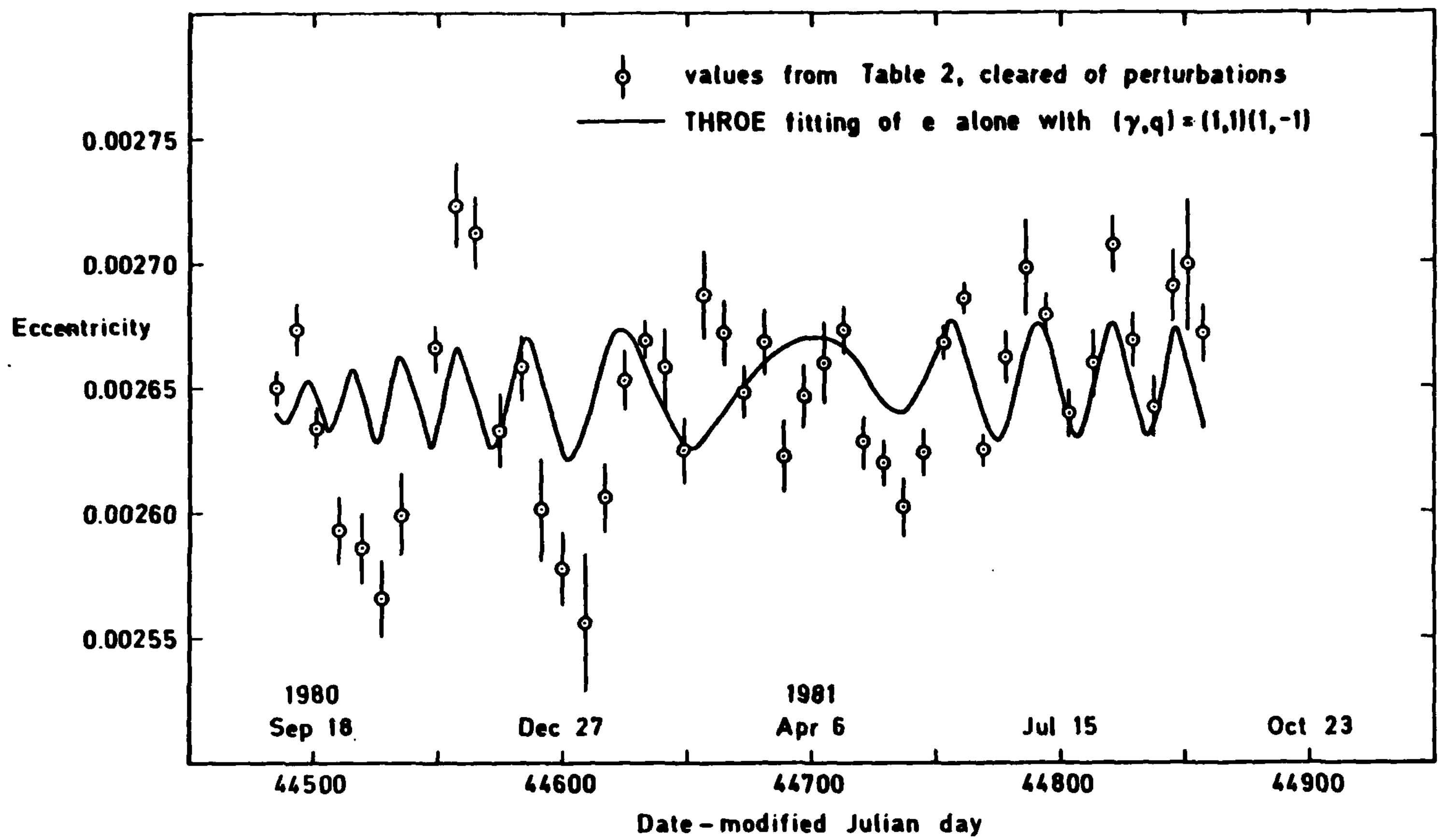

Fig 5 Values of eccentricity near 15th-order resonance, with fitted theoretical curve

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	RAE TR 84008	N/A	UNCLASSIFIED

5. DRIC Code for Originator	6. Originator (Corporate Author) Name and Location
7673000W	Royal Aircraft Establishment, Farnborough, Hants, UK
5a. Sponsoring Agency's Code	**6a. Sponsoring Agency (Contract Authority) Name and Location**
N/A	N/A

7. Title Geopotential harmonics of order 15 and 30, from analysis of the orbit of satellite 1971-10B

7a. (For Translations) Title in Foreign Language

7b. (For Conference Papers) Title, Place and Date of Conference

8. Author 1. Surname, Initials	9a. Author 2	9b. Authors 3, 4	10. Date	Pages	Refs.
Walker, Doreen M.C.			January 1984	24	23

11. Contract Number	12. Period	13. Project	14. Other Reference Nos.
N/A	N/A		Space 633

15. Distribution statement

(a) Controlled by –

(b) Special limitations (if any) –

16. Descriptors (Keywords) (Descriptors marked * are selected from TEST)

Orbit determination. Orbit analysis. Geopotential harmonics. Satellite orbits. Resonance.

17. Abstract

The satellite 1971-10B passed through exact 15th-order resonance on 1981 March 30, and orbital parameters have been determined at 52 epochs from some 3500 observations using the RAE orbit refinement program, PROP, between 1980 September and 1981 October.

The variations in inclination and eccentricity during this time have been analysed, and six lumped 15th-order harmonic coefficients and two 30th-order coefficients have been evaluated. The 15th-order coefficients are the best yet obtained for an orbital inclination near 65°; and previously there were no 30th-order coefficients available at this inclination.

The lumped coefficients have been used to test the Goddard Earth Model GEM 10B: there is good agreement for seven of the eight coefficients.

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